K-THEORY FOR THE SIMPLE C^*-ALGEBRA OF THE FIBONACCHI DYCK SYSTEM

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ABSTRACT. Let F be the Fibonacci matrix $\left[\begin{smallmatrix} 1 & 1 \\ 1 & 0 \end{smallmatrix}\right]$. The Fibonacci Dyck shift is a subshsystem of the Dyck shift D_2 constrained by the matrix F. Let $\mathfrak{L}^{Ch(D_F)}$ be a λ-graph system presenting the subshift D_F, that is called the Cantor horizon λ-graph system for D_F. We will study the C^*-algebra $\mathcal{O}_{\mathfrak{L}^{Ch(D_F)}}$ associated with $\mathfrak{L}^{Ch(D_F)}$. It is simple purely infinite and generated by four partial isometries with some operator relations. We will compute the K-theory of the C^*-algebra. As a result, the C^*-algebra is simple purely infinite and not semiprojective. Hence it is not stably isomorphic to any Cuntz-Krieger algebra.

Keywords: C^*-algebra, Cuntz-Krieger algebra, subshift, λ-graph system, Dyck shift, K-theory,

Mathematics Subject Classification 2000: Primary 46L80; Secondary 46L55, 37B10.

1. Introduction

Let Σ be a finite set with its discrete topology, that is called an alphabet. Each element of Σ is called a symbol. Let $\Sigma^{\mathbb{Z}}$ be the infinite product space $\prod_{i=-\infty}^{\infty} \Sigma_i$, where $\Sigma_i = \Sigma$, endowed with the product topology. The transformation σ on $\Sigma^{\mathbb{Z}}$ given by $\sigma((x_i)_{i\in\mathbb{Z}}) = (x_{i+1})_{i\in\mathbb{Z}}$ is called the full shift over Σ. Let Λ be a closed subset of $\Sigma^{\mathbb{Z}}$ such that $\sigma(\Lambda) = \Lambda$. The topological dynamical system $(\Lambda, \sigma|_\Lambda)$ is called a subshift or a symbolic dynamical system. It is written as Λ for brevity.

In [17], the author has introduced a notion of λ-graph system as a presentation of subshifts. A λ-graph system $\mathfrak{L} = (V, E, \lambda, \iota)$ consists of a vertex set $V = V_0 \cup V_1 \cup V_2 \cup \cdots$, an edge set $E = E_{0,1} \cup E_{1,2} \cup E_{2,3} \cup \cdots$, a labeling map $\lambda : E \to \Sigma$ and a surjective map $\iota_{l,l+1} : V_{l+1} \to V_l$ for each $l \in \mathbb{Z}_+$, where $\mathbb{Z}_+$ denotes the set of all nonnegative integers. An edge $e \in E_{l,l+1}$ has its source vertex $s(e)$ in V_l, its terminal vertex $t(e)$ in V_{l+1} and its label $\lambda(e)$ in Σ ([17]).

The theory of symbolic dynamical system has a close relationship to automata theory and language theory. In the theory of language, there is a class of universal languages due to W. Dyck. The symbolic dynamics generated by the languages are called the Dyck shifts D_N (cf. [3], [10],[11],[12]). Its alphabet consists of the $2N$ brackets: $(_1, \dots, (_N,)_1, \dots,)_N$. The forbidden words consist of words that do not obey the standard bracket rules. In [14], a λ-graph system $\mathfrak{L}^{Ch(D_N)}$ that presents the subshift D_N has been introduced. The λ-graph system is called the Cantor horizon λ-graph system for the Dyck shift D_N. The K-groups for $\mathfrak{L}^{Ch(D_N)}$, that are invariant under topological conjugacy of the subshift D_N, have been computed ([14]).

In [22] (cf. [14]), the C^*-algebra $\mathcal{O}_{\mathfrak{L}^{Ch(D_N)}}$ associated with the Cantor horizon λ-graph system $\mathfrak{L}^{Ch(D_N)}$ has been studied. In the paper, it has been proved that the algebra $\mathcal{O}_{\mathfrak{L}^{Ch(D_N)}}$ is simple and purely infinite and generated by N partial isometries and N isometries satisfying some operator relations. Its K-groups are

$$K_0(\mathcal{O}_{\mathfrak{L}^{Ch(D_N)}}) \cong \mathbb{Z}/N\mathbb{Z} \oplus C(\mathfrak{K}, \mathbb{Z}), \qquad K_1(\mathcal{O}_{\mathfrak{L}^{Ch(D_N)}}) \cong 0$$

where $C(\mathfrak{K}, \mathbb{Z})$ denotes the abelian group of all integer valued continuous functions on a Cantor discontinuum $\mathfrak{K}$ (cf. [14]).

Let $u_1, \dots, u_N$ be the canonical generating isometries of the Cuntz algebra $\mathcal{O}_N$ that satisfy the relations: $\sum_{j=1}^N u_j u_j^* = 1$, $u_i^* u_i = 1$ for $i = 1, \dots N$. Then the bracket rule of the symbols $(_1, \dots, (_N,)_1, \dots,)_N$ of the Dyck shift D_N may be interpreted as the relations $u_i^* u_i = 1$, $u_i^* u_j = 0$ for $i \neq j$ of the partial isometries $u_1^*, \dots, u_N^*, u_1, \dots, u_N$ in the C^*-algebra $\mathcal{O}_N$ (cf. (2.1)).

In [23], we have considered a generalization of Dyck shifts D_N by using the canonical generators of the Cuntz-Krieger algebras $\mathcal{O}_A$ for $N \times N$ matrices A with entries in $\{0, 1\}$. The generalized Dyck shift is denoted by D_A and called the topological Markov Dyck shift for A (cf. [7], [15]). Let $\alpha_1, \dots, \alpha_N$, $\beta_1, \dots, \beta_N$ be the alphabet of D_A. They correspond to the brackets $(_1, \dots, (_N,)_1, \dots,)_N$ respectively. Let $t_1, \dots, t_N$ be the canonical generating partial isometries of the Cuntz-Krieger algebra $\mathcal{O}_A$ that satisfy the relations: $\sum_{j=1}^N t_j t_j^* = 1$, $t_i^* t_i = \sum_{j=1}^N A(i,j) t_j t_j^*$ for $i = 1, \dots, N$. Consider the correspondence $\varphi(\alpha_i) = t_i^*, \varphi(\beta_i) = t_i, i = 1, \dots, N$. Then a word w of $\{\alpha_1, \dots, \alpha_N, \beta_1, \dots, \beta_N\}$ is defined to be admissible for the subshift D_A precisely if the correspnoding element to w through φ in $\mathcal{O}_A$ is not zero. Hence we may recognize D_A to be the subshift defined by the canonical generators of the Cuntz-Krieger algebra $\mathcal{O}_A$. The subshifts D_A are not sofic in general and reduced to the Dyck shifts if all entries of A are 1.

The Cantor horizon λ-graph system $\mathfrak{L}^{Ch(D_A)}$ for the topological Markov Dyck shift D_A has been also studied in [23]. It has been proved to be λ-irreducible with λ-condition (I) in the sense of [21] if the matrix is irreducible with condition (I) in the sense of Cuntz-Krieger [5]. Hence the associated C^*-algebra $\mathcal{O}_{\mathfrak{L}^{Ch(D_A)}}$ is simple and purely infinite. It is the unique C^*-algebra generated by $2N$ partial isometries subject to some operator relations.

In this paper we study the C^*-algebra $\mathcal{O}_{\mathfrak{L}^{Ch(D_F)}}$ for the Fibonacci matrix $F = \left[\begin{smallmatrix} 1 & 1 \\ 1 & 0 \end{smallmatrix}\right]$. It is the smallest matrix in the irreducible square matices with condition (I) such that the associated topological Markov shift Λ_F is not conjugate to any full shift. The topological entropy of Λ_F is $\log \frac{1+\sqrt{5}}{2}$ the logarithm of the Perron eigenvalue of F. We call the subshift D_F the Fibonacci Dyck shift. As the matrix is irreducible with condition (I), the associated C^*-algebra $\mathcal{O}_{\mathfrak{L}^{Ch(D_F)}}$ is simple and purely infinite. We will compute the K-groups $K_i(\mathcal{O}_{\mathfrak{L}^{Ch(D_F)}}), i = 0, 1$ of the algebra so that we have

Theorem 1.1. *The C^*-algebra $\mathcal{O}_{\mathfrak{L}^{Ch(D_F)}}$ associated with the λ-graph system $\mathfrak{L}^{Ch(D_F)}$ is unital, separable, nuclear, simple and purely infinite. It is the unique C^*-algebra generated by one isometry T_1 and three partial isometries S_1, S_2, T_2 subject to the following operator relations:*

$$\sum_{j=1}^{2} (S_j S_j^* + T_j T_j^*) = \sum_{j=1}^{2} S_j^* S_j = 1, \qquad T_2^* T_2 = S_1^* S_1, \tag{1.1}$$

$$E_{\mu_1\cdots\mu_k} = \sum_{j=1}^{2} F(j,\mu_1) S_j S_j^* E_{\mu_1\cdots\mu_k} S_j S_j^* + T_{\mu_1} E_{\mu_2\cdots\mu_k} T_{\mu_1}^*, \qquad k > 1 \tag{1.2}$$

where $E_{\mu_1\cdots\mu_k} = S_{\mu_1}^* \cdots S_{\mu_k}^* S_{\mu_k} \cdots S_{\mu_1}$, $(\mu_1, \cdots, \mu_k) \in \Lambda_F^*$, *and* Λ_F^* *is the set of admissible words of the topological Markov shift* Λ_F *defined by the matrix* F. *The* K*-groups are*

$$K_0(\mathcal{O}_{\mathfrak{L}^{Ch(D_F)}}) \cong \mathbb{Z} \oplus C(\mathfrak{K}, \mathbb{Z})^\infty, \qquad K_1(\mathcal{O}_{\mathfrak{L}^{Ch(D_F)}}) \cong 0.$$

This paper is a continuation of [23].

2. The subshift D_A and the λ-graph system $\mathfrak{L}^{Ch(D_A)}$

We will briefly review the topological Markov Dyck shift D_A and its Cantor horizon λ-graph system $\mathfrak{L}^{Ch(D_A)}$.

Consider a pair of N symbols where $\Sigma^- = \{\alpha_1, \cdots, \alpha_N\}, \Sigma^+ = \{\beta_1, \cdots, \beta_N\}$. We set $\Sigma = \Sigma^- \cup \Sigma^+$. Let $A = [A(i,j)]_{i,j=1,\dots,N}$ be an $N \times N$ matrix with entries in $\{0,1\}$. Throughout this paper, A is assumed to have no zero rows or columns. Consider the Cuntz-Krieger algebra $\mathcal{O}_A$ for the matrix A that is the universal C^*-algebra generated by N partial isometries $t_1, \dots, t_N$ subject to the following relations:

$$\sum_{j=1}^{N} t_j t_j^* = 1, \qquad t_i^* t_i = \sum_{j=1}^{N} A(i,j) t_j t_j^* \quad \text{for } i = 1, \dots, N$$

([5]). Define a correspondence $\varphi_A : \Sigma \longrightarrow \{t_1^*, \dots, t_N^*, t_1, \dots, t_N\}$ by setting

$$\varphi_A(\alpha_i) = t_i^*, \qquad \varphi_A(\beta_i) = t_i \quad \text{for } i = 1, \dots, N.$$

We denote by Σ^* the set of all words $\gamma_1 \cdots \gamma_n$ of elements of Σ. Define the set

$$\mathfrak{F}_A = \{\gamma_1 \cdots \gamma_n \in \Sigma^* \mid \varphi_A(\gamma_1) \cdots \varphi_A(\gamma_n) = 0\}.$$

Let D_A be the subshift over Σ whose forbidden words are $\mathfrak{F}_A$. The subshift is called the topological Markov Dyck shift defined by A (cf. [7], [15]). If all entries of A are 1, the subshift D_A becomes the Dyck shift D_N with $2N$ bracket (cf. [11],[12], [14], [22],[23]). We note the fact that $\alpha_i \beta_j \in \mathfrak{F}_A$ if $i \neq j$, and $\alpha_{i_n} \cdots \alpha_{i_1} \in \mathfrak{F}_A$ if and only if $\beta_{i_1} \cdots \beta_{i_n} \in \mathfrak{F}_A$. Consider the following subsystem of D_A

$$D_A^+ = \{(\gamma_i)_{i \in \mathbb{Z}} \in D_A \mid \gamma_i \in \Sigma^+ \text{ for all } i \in \mathbb{Z}\}.$$

The subshift D_A^+ is identified with the topological Markov shift

$$\Lambda_A = \{(x_i)_{i \in \mathbb{Z}} \in \{1, \dots, N\}^{\mathbb{Z}} \mid A(x_i, x_{i+1}) = 1, i \in \mathbb{Z}\}$$

defined by the matrix A. Hence the subshift D_A is recognized to contain the topological Markov shift Λ_A.

We denote by $B_l(D_A)$ and $B_l(\Lambda_A)$ the set of admissible words of length l of D_A and that of Λ_A respectively. Let $m(l)$ be the cardinal number of $B_l(\Lambda_A)$. We use lexcographic order from the left on the words of $B_l(\Lambda_A)$, so that we may assign to a word $\mu_1 \cdots \mu_l \in B_l(\Lambda_A)$ the number $N(\mu_1 \cdots \mu_l)$ from 1 to $m(l)$. For example, if $A = F = \left[\begin{smallmatrix} 1 & 1 \\ 1 & 0 \end{smallmatrix}\right]$, then

$$B_1(\Lambda_F) = \{1, 2\}, \qquad N(1) = 1,\ N(2) = 2,$$
$$B_2(\Lambda_F) = \{11, 12, 21\}, \qquad N(11) = 1,\ N(12) = 2,\ N(21) = 3,$$

and so on. Hence the set $B_l(\Lambda_A)$ bijectively corresponds to the set of natural numbers less than or equal to $m(l)$. Let us now describe the Cantor horizon λ-graph system $\mathfrak{L}^{Ch(D_A)}$ of D_A. The vertices V_l at level l for $l \in \mathbb{Z}_+$ are given by the admissible words of length l consisting of the symbols of Σ^+. We regard V_0 as a one point set of the empty word $\{\emptyset\}$. Since V_l is identified with $B_l(\Lambda_A)$, we may write V_l as

$$V_l = \{v^l_{N(\mu_1\cdots\mu_l)} \mid \mu_1\cdots\mu_l \in B_l(\Lambda_A)\}.$$

The mapping $\iota(=\iota_{l,l+1}) : V_{l+1} \to V_l$ is defined by deleting the rightmost symbol of a corresponding word such as

$$\iota(v^{l+1}_{N(\mu_1\cdots\mu_{l+1})}) = v^l_{N(\mu_1\cdots\mu_l)} \qquad \text{for} \qquad v^{l+1}_{N(\mu_1\cdots\mu_{l+1})} \in V_{l+1}.$$

We define an edge labeled α_j from $v^l_{N(\mu_1\cdots\mu_l)} \in V_l$ to $v^{l+1}_{N(\mu_0\mu_1\cdots\mu_l)} \in V_{l+1}$ precisely if $\mu_0 = j$, and an edge labeled β_j from $v^l_{N(j\mu_1\cdots\mu_{l-1})} \in V_l$ to $v^{l+1}_{N(\mu_1\cdots\mu_{l+1})} \in V_{l+1}$. For $l = 0$, we define an edge labeled α_j form v^0_1 to $v^1_{N(j)}$, and an edge labeled β_j form v^0_1 to $v^1_{N(i)}$ if $A(j,i) = 1$. We denote by $E_{l,l+1}$ the set of edges from V_l to V_{l+1}. Set $E = \cup_{l=0}^{\infty} E_{l,l+1}$. It is easy to see that the resulting labeled Bratteli diagram with ι-map becomes a λ-graph system over Σ, that is called the Cantor horizon Λ-graph system and is denoted by $\mathfrak{L}^{Ch(D_A)}$.

A λ-graph system $\mathfrak{L}$ is said to present a subshift Λ if the set of all admissible words of Λ coincides with the set of all finite labeled sequences appearing in concatenating edges of $\mathfrak{L}$. In [23], the following propositions have been proved.

Proposition 2.1.

(i) *If A satisfies condition (I) in the sense of Cuntz-Krieger* [5]*, the subshift D_A is not sofic.*

(ii) *The λ-graph system $\mathfrak{L}^{Ch(D_A)}$ presents the subshift D_A.*

(iii) *If A is an irreducible matrix with condition (I), then the λ-graph system $\mathfrak{L}^{Ch(D_A)}$ is λ-irreducible with λ-condition (I) in the sense of* [21].

Proposition 2.2. *The C^*-algebra $\mathcal{O}_{\mathfrak{L}^{Ch(D_A)}}$ associated with the λ-graph system $\mathfrak{L}^{Ch(D_A)}$ is unital, separable, nuclear, simple and purely infinite. It is the unique C^*-algebra generated by $2N$ partial isometries $S_i, T_i, i = 1, \dots, N$ subject to the following operator relations:*

$$\sum_{j=1}^{N}(S_jS_j^* + T_jT_j^*) = \sum_{j=1}^{N} S_j^*S_j = 1,$$

$$T_i^*T_i = \sum_{j=1}^{N} A(i,j)S_j^*S_j, \qquad i = 1, 2, \dots, N,$$

$$E_{\mu_1\cdots\mu_k} = \sum_{j=1}^{N} A(j,\mu_1)S_jS_j^*E_{\mu_1\cdots\mu_k}S_jS_j^* + T_{\mu_1}E_{\mu_2\cdots\mu_k}T_{\mu_1}^*, \qquad k > 1$$

where $E_{\mu_1\cdots\mu_k} = S_{\mu_1}^*\cdots S_{\mu_k}^*S_{\mu_k}\cdots S_{\mu_1}$, $(\mu_1,\cdots,\mu_k) \in \Lambda_A^*$ the set of admissible words of the topological Markov shift Λ_A defined by the matrix A.

3. K-theory for $\mathcal{O}_{\mathfrak{L}^{Ch(D_F)}}$

We will prove Theorem 1.1. The operator relations (1.1) and (1.2) are direct from the operator relations in Proposition 2.2. By Proposition 2.2, it remains to

prove the K-group formulae. This section is devoted to computing the K-groups $K_i(\mathcal{O}_{\mathfrak{L}^{Ch(D_F)}}), i = 0, 1$ for the C^*-algebra $\mathcal{O}_{\mathfrak{L}^{Ch(D_F)}}$. The symbols α_1, α_2, β_1, β_2 of the subshift D_F correspond to the brackets $(_1$, $(_2$, $)_1$, $)_2$ respectively. Let $V_l, l \in \mathbb{Z}$ be the vertex set of the λ-graph system $\mathfrak{L}^{Ch(D_F)}$. They are identified with the admissible words consisting of the symbols β_1, β_2 in Σ^+. Since the word $\beta_2\beta_2$ is forbidden, the following is a list of the vertex sets V_l for $l = 0, 1, 2, 3, 4, \dots$:

$$
\begin{aligned}
V_0 &: * \\
V_1 &: (\beta_1),\ (\beta_2), \\
V_2 &: (\beta_1\beta_1),\ (\beta_1\beta_2),\ (\beta_2\beta_1), \\
V_3 &: (\beta_1\beta_1\beta_1),\ (\beta_1\beta_1\beta_2),\ (\beta_1\beta_2\beta_1),\ (\beta_2\beta_1\beta_1),\ (\beta_2\beta_1\beta_2), \\
V_4 &: (\beta_1\beta_1\beta_1\beta_1),\ (\beta_1\beta_1\beta_1\beta_2),\ (\beta_1\beta_1\beta_2\beta_1),\ (\beta_1\beta_2\beta_1\beta_1),\ (\beta_1\beta_2\beta_1\beta_2), \\
&\quad (\beta_2\beta_1\beta_1\beta_1),\ (\beta_2\beta_1\beta_1\beta_2),\ (\beta_2\beta_1\beta_2\beta_1), \\
&\quad \cdots
\end{aligned}
$$

Let f_l be the l-th Fibonacci number for $l \in \mathbb{N}$. They are inductively defined by

$$f_1 = f_2 = 1, \qquad f_{l+2} = f_{l+1} + f_l \quad \text{for } l \in \mathbb{N}.$$

By the structure of the λ-graph system $\mathfrak{L}^{Ch(D_F)}$, the number $m(l)$ of the vertex set V_l is f_{l+2}. We denote by $(\mathcal{M}_{l,l+1}, I_{l,l+1})_{l\in\mathbb{Z}_+}$ the symbolic matrix system of the Cantor horizon λ-graph system $\mathfrak{L}^{Ch(D_F)}$. We write the vertex set V_l as $\{v_1^l, \dots, v_{m(l)}^l\}$. Both the matrices $\mathcal{M}_{l,l+1}$ and $I_{l,l+1}$ are the $m(l)\times m(l+1)$ matrices for each $l \in \mathbb{Z}_+$. For $i = 1, \dots, m(l)$, $j = 1, \dots, m(l+1)$, the component $\mathcal{M}_{l,l+1}(i,j)$ denotes the formal sum of labels of edges starting at the vertex v_i^l and terminating at the vertex v_j^{l+1}, and the component $I_{l,l+1}(i,j)$ denotes 1 if $\iota(v_j^{l+1} = v_i^l$, otherwise 0. They satisfy the relations $I_{l,l+1}\mathcal{M}_{l+1,l+2} = \mathcal{M}_{l,l+1}I_{l+1,l+2}$ for $l \in \mathbb{Z}_+$ as symbolic matrices. The orderings of the rows and columns of the matrices are arranged lexcographically on indices $i_1 \cdots i_n$ of the words $\beta_{i_1} \cdots \beta_{i_n}$ from the left. Let us denote by $0_{p,q}$ the $m(p) \times m(q)$ matrix all of whose entries are $0'$s.

Lemma 3.1. *The $m(l) \times m(l+1)$ matrix $I_{l,l+1}$ is given by :*

$$I_{0,1} = \begin{bmatrix} 1 & 1 \end{bmatrix}, \qquad I_{1,2} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad I_{l+2,l+3} = \begin{bmatrix} I_{l+1,l+2} & 0_{l+1,l+1} \\ 0_{l,l+2} & I_{l,l+1} \end{bmatrix}, \quad l \in \mathbb{Z}_+.$$

In what follows, blanks at components of matrices denote 0's. For $l \in \mathbb{Z}_+$ and $a \in \{\alpha_1, \alpha_2, \beta_1, \beta_2\}$, let $I_l(a)$ be the $m(l) \times m(l)$ diagonal matrix with diagonal entries a, and $\mathcal{S}_l(a)$ the $m(l-1) \times m(l+1)$ matrix defined by

$$\mathcal{S}_0(a) = \begin{bmatrix} a & a \end{bmatrix}, \quad \mathcal{S}_1(a) = \begin{bmatrix} a & a & a \end{bmatrix}, \quad \mathcal{S}_{l+2}(a) = \begin{bmatrix} \mathcal{S}_{l+1}(a) & 0_{l,l+1} \\ 0_{l-1,l+2} & \mathcal{S}_l(a) \end{bmatrix}$$

where $m(-1)$ denotes 1. For $l = 2, 3, 4$ and $a \in \{\alpha_1, \alpha_2, \beta_1, \beta_2\}$, one sees that

$$\mathcal{S}_2(a) = \begin{bmatrix} a & a & a & & \\ & & & a & a \end{bmatrix} \quad : 2 \times 5 \text{ matrix},$$

$$\mathcal{S}_3(a) = \begin{bmatrix} a & a & a & & & & & \\ & & & a & a & & & \\ & & & & & a & a & a \end{bmatrix} \quad : 3 \times 8 \text{ matrix},$$

and

$$\mathcal{S}_4(a) = \begin{bmatrix} a & a & a & & & & & & & & & & \\ & & & a & a & & & & & & & & \\ & & & & & a & a & a & & & & & \\ & & & & & & & & a & a & a & & \\ & & & & & & & & & & & a & a \end{bmatrix} \quad : 5 \times 13 \text{ matrix.}$$

Lemma 3.2. *The $m(l) \times m(l+1)$ matrix $\mathcal{M}_{l,l+1}$ is given by :*

$$\mathcal{M}_{0,1} = [\alpha_1 + \beta_1 + \beta_2,\ \alpha_2 + \beta_1],$$

$$\mathcal{M}_{l,l+1} = \left[\begin{array}{c|c} \multicolumn{2}{c}{\mathcal{S}_l(\beta_1)} \\ \hline \mathcal{S}_{l-1}(\beta_2) & 0_{l-2,l-1} \end{array}\right] + \left[\begin{array}{c|c} I_l(\alpha_1) & \begin{array}{c} I_{l-1}(\alpha_2) \\ \hline 0_{l-2,l-1} \end{array} \end{array}\right].$$

Proof. In the right hand side in the second equation above, the first summand describes the transitions that arise when a vertex accepts a symbol in Σ^+. The second summand describes the transitions that arise when a vertex accepts a symbol in Σ^-. □

We present the above matrices for $l = 1, 2, 3, 4$:

$$I_{1,2} = \begin{bmatrix} 1 & 1 & \\ & & 1 \end{bmatrix}, \qquad \mathcal{M}_{1,2} = \begin{bmatrix} \alpha_1+\beta_1 & \beta_1 & \alpha_2+\beta_1 \\ \beta_2 & \alpha_1+\beta_2 & \end{bmatrix},$$

$$I_{2,3} = \begin{bmatrix} 1 & 1 & & & \\ & & 1 & & \\ & & & 1 & 1 \end{bmatrix}, \qquad \mathcal{M}_{2,3} = \begin{bmatrix} \alpha_1+\beta_1 & \beta_1 & \beta_1 & \alpha_2 & \\ & \alpha_1 & & \beta_1 & \alpha_2+\beta_1 \\ \beta_2 & \beta_2 & \alpha_1+\beta_2 & & \end{bmatrix},$$

$$I_{3,4} = \begin{bmatrix} 1 & 1 & & & & & & \\ & & 1 & & & & & \\ & & & 1 & 1 & & & \\ & & & & & 1 & 1 & \\ & & & & & & & 1 \end{bmatrix}, \qquad \mathcal{M}_{3,4} = \begin{bmatrix} \alpha_1+\beta_1 & \beta_1 & \beta_1 & & & \alpha_2 & & \\ & \alpha_1 & & \beta_1 & \beta_1 & & \alpha_2 & \\ & & \alpha_1 & & & \beta_1 & \beta_1 & \alpha_2+\beta_1 \\ \beta_2 & \beta_2 & \beta_2 & \alpha_1 & & & & \\ & & & \beta_2 & \alpha_1+\beta_2 & & & \end{bmatrix},$$

$$I_{4,5} = \begin{bmatrix} 1 & 1 & & & & & & & & & & & \\ & & 1 & & & & & & & & & & \\ & & & 1 & 1 & & & & & & & & \\ & & & & & 1 & 1 & & & & & & \\ & & & & & & & 1 & & & & & \\ & & & & & & & & 1 & 1 & & & \\ & & & & & & & & & & 1 & & \\ & & & & & & & & & & & 1 & 1 \end{bmatrix},$$

$$\mathcal{M}_{4,5} = \begin{bmatrix} \alpha_1+\beta_1 & \beta_1 & \beta_1 & & & & & & \alpha_2 & & & & \\ & \alpha_1 & & \beta_1 & \beta_1 & & & & & \alpha_2 & & & \\ & & \alpha_1 & & & \beta_1 & \beta_1 & \beta_1 & & & \alpha_2 & & \\ & & & \alpha_1 & & & & & \beta_1 & \beta_1 & \beta_1 & \alpha_2 & \\ & & & & \alpha_1 & & & & & & & \beta_1 & \beta_1+\alpha_2 \\ \beta_2 & \beta_2 & \beta_2 & & & \alpha_1 & & & & & & & \\ & & & \beta_2 & \beta_2 & & \alpha_1 & & & & & & \\ & & & & & \beta_2 & \beta_2 & \beta_2+\alpha_1 & & & & & \end{bmatrix}.$$

Let $(M_{l,l+1}, I_{l,l+1})_{l \in \mathbb{Z}_+}$ be the nonnegative matrix system for $(\mathcal{M}, I)$. The matrix $M_{l,l+1}$ for each $l \in \mathbb{Z}_+$ is obtained from $\mathcal{M}_{l,l+1}$ by setting all the symbols of $\mathcal{M}_{l,l+1}$ equal to 1. That is, the (i,j)-component $M_{l,l+1}(i,j)$ of the matrix $M_{l,l+1}$ denotes the number of the symbols in Σ that appear in $\mathcal{M}_{l,l+1}(i,j)$. The groups $K_0(\mathcal{O}_{\mathfrak{L}^{Ch(D_F)}}), K_1(\mathcal{O}_{\mathfrak{L}^{Ch(D_F)}})$ are realized as the K-groups $K_0(M,I)$ and $K_1(M,I)$ for the nonnegative matrix system (M,I) respectively (cf. [18]). They are calculated by the following formulae.

Lemma 3.3 (([18], cf. [17]))**.**

(i) $K_0(\mathcal{O}_{\mathfrak{L}^{Ch(D_F)}}) = \varinjlim_l \{\mathbb{Z}^{m(l+1)}/(M^t_{l,l+1} - I^t_{l,l+1})\mathbb{Z}^{m(l)}, \bar{I}^t_{l,l+1}\}$, *where the inductive limit is taken along the natural induced homomorphisms $\bar{I}^t_{l,l+1}, l \in \mathbb{Z}_+$ by the matrices $I^t_{l,l+1}$.*

(ii) $K_1(\mathcal{O}_{\mathfrak{L}^{Ch(D_F)}}) = \varinjlim_{l}\{\mathrm{Ker}(M^t_{l,l+1} - I^t_{l,l+1})$ *in* $\mathbb{Z}^{m(l)}, I^t_{l,l+1}\}$, *where the inductive limit is taken along the homomorphisms of the restrictions of* $I^t_{l,l+1}$ *to* $\mathrm{Ker}(M^t_{l,l+1} - I^t_{l,l+1})$.

By the formulae of $\mathcal{M}_{l,l+1}$ in Lemma 3.2, the matrices $M^t_{l,l+1} - I^t_{l,l+1}$ for $l = 1, 2, 3, 4$ are presented as in the following way:

$$M^t_{1,2} - I^t_{1,2} = \left[\begin{array}{c|c} 2 & 1 \\ 1 & 2 \\ \hline 2 & \end{array}\right] - \left[\begin{array}{c|c} 1 & \\ 1 & \\ \hline & 1 \end{array}\right] = \left[\begin{array}{c|c} 1 & 1 \\ & 2 \\ \hline 2 & -1 \end{array}\right],$$

$$M^t_{2,3} - I^t_{2,3} = \left[\begin{array}{cc|c} 2 & & 1 \\ 1 & 1 & 1 \\ 1 & & 2 \\ \hline 1 & 1 & \\ & 2 & \end{array}\right] - \left[\begin{array}{cc|c} 1 & & \\ 1 & & \\ & 1 & \\ \hline & & 1 \\ & & 1 \end{array}\right] = \left[\begin{array}{cc|c} 1 & & 1 \\ & 1 & 1 \\ 1 & -1 & 2 \\ \hline 1 & 1 & -1 \\ & 2 & -1 \end{array}\right],$$

$$M^t_{3,4} - I^t_{3,4} = \left[\begin{array}{ccc|cc} 2 & & & 1 & \\ 1 & 1 & & 1 & \\ 1 & & 1 & 1 & \\ & 1 & & 1 & 1 \\ & 1 & & & 2 \\ \hline 1 & & 1 & & \\ & 1 & 1 & & \\ & & 2 & & \end{array}\right] - \left[\begin{array}{ccc|cc} 1 & & & & \\ 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & 1 & & \\ \hline & & & 1 & \\ & & & 1 & \\ & & & & 1 \end{array}\right] = \left[\begin{array}{ccc|cc} 1 & & & 1 & \\ & 1 & & 1 & \\ 1 & -1 & 1 & 1 & \\ & 1 & -1 & 1 & 1 \\ & 1 & -1 & & 2 \\ \hline 1 & & 1 & -1 & \\ & 1 & 1 & -1 & \\ & & 2 & & -1 \end{array}\right],$$

and

$$M^t_{4,5} - I^t_{4,5} = \left[\begin{array}{ccccc|ccc} 2 & & & & & 1 & & \\ 1 & 1 & & & & 1 & & \\ 1 & & 1 & & & 1 & & \\ & 1 & & 1 & & & 1 & \\ & 1 & & & 1 & & 1 & \\ & & 1 & & & 1 & & 1 \\ & & 1 & & & & 1 & 1 \\ & & 1 & & & & & 2 \\ \hline 1 & & & 1 & & & & \\ & 1 & & 1 & & & & \\ & & 1 & 1 & & & & \\ & & & 1 & 1 & & & \\ & & & & 2 & & & \end{array}\right] - \left[\begin{array}{ccccc|ccc} 1 & & & & & & & \\ 1 & & & & & & & \\ & 1 & & & & & & \\ & & 1 & & & & & \\ & & 1 & & & & & \\ & & & 1 & & & & \\ & & & 1 & & & & \\ & & & & 1 & & & \\ \hline & & & & & 1 & & \\ & & & & & 1 & & \\ & & & & & & 1 & \\ & & & & & & & 1 \\ & & & & & & & 1 \end{array}\right]$$

$$
= \left[\begin{array}{ccccc|ccc}
1 & & & & & 1 & & \\
& 1 & & & & 1 & & \\
1 & -1 & 1 & & & 1 & & \\
& 1 & -1 & 1 & & & 1 & \\
& 1 & -1 & & 1 & & 1 & \\
& & 1 & -1 & & 1 & & 1 \\
& & 1 & -1 & & & 1 & 1 \\
& & 1 & & -1 & & & 2 \\
\hline
1 & & & 1 & & -1 & & \\
& 1 & & 1 & & -1 & & \\
& & 1 & 1 & & & -1 & \\
& & & 1 & 1 & & & -1 \\
& & & & 2 & & & -1
\end{array}\right].
$$

It is easy to see that the kernels of the matrices $M^t_{l,l+1} - I^t_{l,l+1}$ are $\{0\}$ for all $l \in \mathbb{N}$. Hence $K_1(\mathcal{O}_{\mathfrak{L}^{Ch(D_F)}}) = \{0\}$ is obvious. The computation of the K_0-group $K_0(\mathcal{O}_{\mathfrak{L}^{Ch(D_F)}})$ is the main body of this section. We denote by $\mathbb{A}_{l+1,l}$ the $m(l+1)\times m(l)$ matrix $M^t_{l,l+1} - I^t_{l,l+1}$. We will compute the cokernels of the matrices $\mathbb{A}_{l+1,l}$. We set subblock matrices $\mathbb{A}^{UL}_{l+1,l}, \mathbb{A}^{UR}_{l+1,l}, \mathbb{A}^{LL}_{l+1,l}$ and $\mathbb{A}^{LR}_{l+1,l}$ of $\mathbb{A}_{l+1,l}$ by setting

$$
\begin{aligned}
&\mathbb{A}^{UL}_{l+1,l}(i,j) = \mathbb{A}_{l+1,l}(i,j) \quad \text{for } 1 \le i \le m(l), 1 \le j \le m(l-1),\\
&\mathbb{A}^{UR}_{l+1,l}(i,j) = \mathbb{A}_{l+1,l}(i, m(l-1)+j) \quad \text{for } 1 \le i \le m(l), 1 \le j \le m(l-2),\\
&\mathbb{A}^{LL}_{l+1,l}(i,j) = \mathbb{A}_{l+1,l}(m(l)+i, j) \quad \text{for } 1 \le i \le m(l-1), 1 \le j \le m(l-1),\\
&\mathbb{A}^{LR}_{l+1,l}(i,j) = \mathbb{A}_{l+1,l}(m(l)+i, m(l-1)+j) \quad \text{for } 1 \le i \le m(l-1), 1 \le j \le m(l-2).
\end{aligned}
$$

They are an $m(l)\times m(l-1)$ matrix, an $m(l)\times m(l-2)$ matrix, an $m(l-1)\times m(l-1)$ matrix and an $m(l-1)\times m(l-2)$ matrix respectively such that

$$
\mathbb{A}_{l+1,l} = \left[\begin{array}{c|c}
\mathbb{A}^{UL}_{l+1,l} & \mathbb{A}^{UR}_{l+1,l} \\
\hline
\mathbb{A}^{LL}_{l+1,l} & \mathbb{A}^{LR}_{l+1,l}
\end{array}\right].
$$

Let I_l be the $m(l)\times m(l)$ identity matrix. Recall that $0_{k,l}$ denotes the $m(k)\times m(l)$ matrix all of which entries are $0's$. By Lemma 3.1 and Lemma 3.2, one sees the general form of $\mathbb{A}_{l+1,l}$ as in the following way:

Lemma 3.4. *For $l = 3,4,\dots,$, we have*

$$
\mathbb{A}^{UL}_{l+2,l+1} = \left[\begin{array}{cc|c}
\multicolumn{2}{c|}{\mathbb{A}^{UL}_{l+1,l}} & \begin{array}{c} 0_{l-1,l-2} \\ \hline I_{l-2} \end{array} \\
\hline
0_{l-1,l-2} & \multicolumn{1}{|c|}{S^t_{l-2}(1)} & \mathbb{A}^{LR}_{l+1,l}
\end{array}\right],
$$

$$
\mathbb{A}^{UR}_{l+2,l+1} = \left[\begin{array}{c|c}
S^t_{l-1}(1) & 0_{l,l-3} \\
\hline
\multicolumn{2}{c}{\mathbb{A}^{LL}_{l+1,l}}
\end{array}\right],
$$

$$
\mathbb{A}^{LL}_{l+2,l+1} = \left[\begin{array}{c|c}
\begin{array}{c} I_{l-1} \\ \hline 0_{l-2,l-1} \end{array} & \mathbb{A}^{UR}_{l+1,l}
\end{array}\right],
$$

$$
\mathbb{A}^{LR}_{l+2,l+1} = \left[\begin{array}{c|c}
\mathbb{A}^{LR}_{l+1,l} & 0_{l-1,l-3} \\
\hline
0_{l-2,l-2} & \mathbb{A}^{LR}_{l,l-1}
\end{array}\right].
$$

Hence the sequence $\mathbb{A}_{l+1,l}, l \in \mathbb{N}$ of the matrices are inductively determined.

We set the $m(l) \times m(l)$ square matrix B_l by setting

$$B_l = \left[\begin{array}{c|c} \mathbb{A}^{UL}_{l+1,l} & \mathbb{A}^{UR}_{l+1,l} \end{array} \right]$$

the upper half of the matrix $\mathbb{A}_{l+1,l}$. We next provide a sequence $C_{l+1,l}, l \in \mathbb{N}$ of $m(l+1) \times m(l)$ matrix such as:

$$C_{2,1} = \left[\begin{array}{c|c} 2 & \\ 2 & \\ \hline 2 & \end{array} \right], \qquad C_{3,2} \left[\begin{array}{c|c|c} 1 & 1 & \\ 1 & 1 & \\ 1 & 1 & \\ \hline 1 & 1 & \\ 1 & 1 & \end{array} \right], \qquad C_{4,3} = \left[\begin{array}{cc|c|c} 1 & & 1 & \\ 1 & & 1 & \\ 1 & & 1 & \\ 1 & & 1 & \\ 1 & & 1 & \\ \hline & 1 & 1 & \\ & 1 & 1 & \\ & 1 & 1 & \end{array} \right],$$

$$C_{5,4} = \left[\begin{array}{ccc|cc|c} 1 & & & 1 & & \\ 1 & & & 1 & & \\ 1 & & & 1 & & \\ 1 & & & 1 & & \\ 1 & & & 1 & & \\ & 1 & & 1 & & \\ & 1 & & 1 & & \\ & 1 & & 1 & & \\ \hline & & 1 & & 1 & \\ & & 1 & & 1 & \\ & & 1 & & 1 & \\ & & 1 & & 1 & \\ & & 1 & & 1 & \end{array} \right], \qquad C_{6,5} = \left[\begin{array}{ccccc|ccc|c} 1 & & & & & 1 & & & \\ 1 & & & & & 1 & & & \\ 1 & & & & & 1 & & & \\ 1 & & & & & 1 & & & \\ 1 & & & & & 1 & & & \\ & 1 & & & & 1 & & & \\ & 1 & & & & 1 & & & \\ & 1 & & & & 1 & & & \\ & & 1 & & & & 1 & & \\ & & 1 & & & & 1 & & \\ & & 1 & & & & 1 & & \\ & & 1 & & & & 1 & & \\ & & 1 & & & & 1 & & \\ \hline & & & 1 & & & & 1 & \\ & & & 1 & & & & 1 & \\ & & & 1 & & & & 1 & \\ & & & 1 & & & & 1 & \\ & & & 1 & & & & 1 & \\ & & & & 1 & & & 1 & \\ & & & & 1 & & & 1 & \\ & & & & 1 & & & 1 & \end{array} \right].$$

To define the matrices $C_{l+1,l}$ for $l \geq 6$, divide $C_{l+1,l}$ into 6 subblock matrices $C^{UL}_{l+1,l}, C^{UM}_{l+1,l}, C^{UR}_{l+1,l}, C^{LL}_{l+1,l}, C^{LM}_{l+1,l}, C^{LR}_{l+1,l}$ as in the following way:

$$\begin{aligned}
C^{UL}_{l+1,l}(i,j) =& C_{l+1,l}(i,j) \text{ for } 1 \leq i \leq m(l), 1 \leq j \leq m(l-2), \\
C^{UM}_{l+1,l}(i,j) =& C_{l+1,l}(i, j+m(l-2)) \text{ for } 1 \leq i \leq m(l), 1 \leq j \leq m(l-3), \\
C^{UR}_{l+1,l}(i,j) =& C_{l+1,l}(i, j+m(l-2)+m(l-3)) \text{ for } 1 \leq i \leq m(l), 1 \leq j \leq m(l-2), \\
C^{LL}_{l+1,l}(i,j) =& C_{l+1,l}(i+m(l), j) \text{ for } 1 \leq i \leq m(l-1), 1 \leq j \leq m(l-2), \\
C^{LM}_{l+1,l}(i,j) =& C_{l+1,l}(i+m(l), j+m(l-2)) \text{ for } 1 \leq i \leq m(l-1), 1 \leq j \leq m(l-3), \\
C^{LR}_{l+1,l}(i,j) =& C_{l+1,l}(i+m(l), j+m(l-2)+m(l-3)) \text{ for } 1 \leq i \leq m(l-1), 1 \leq j \leq m(l-2).
\end{aligned}$$

They are an $m(l) \times m(l-2)$ matrix, an $m(l) \times m(l-3)$ matrix, an $m(l) \times m(l-2)$ matrix, an $m(l-1) \times m(l-2)$ matrix, an $m(l-1) \times m(l-3)$ matrix and an $m(l-1) \times m(l-2)$ matrix respectively such that

$$C_{l+1,l} = \left[\begin{array}{c|c|c} C^{UL}_{l+1,l} & C^{UM}_{l+1,l} & C^{UR}_{l+1,l} \\ \hline C^{LL}_{l+1,l} & C^{LM}_{l+1,l} & C^{LR}_{l+1,l} \end{array}\right].$$

These block matrices are defined inductively as in the following way:

$$C^{UL}_{l+1,l} = \left[\begin{array}{c|c} C^{UL}_{l,l-1} & \\ \cline{1-1} & 0_{l,l-4} \\ C^{LL}_{l,l-1} & \end{array}\right], \quad C^{UM}_{l+1,l} = \left[\begin{array}{c|c} C^{UM}_{l,l-1} & \\ \cline{1-1} & 0_{l,l-5} \\ C^{LM}_{l,l-1} & \end{array}\right], \quad C^{UL}_{l+1,l} = [0_{l,l-2}],$$

$$C^{LL}_{l+1,l} = \left[\begin{array}{c|c} & C^{UL}_{l-1,l-2} \\ \cline{2-2} 0_{l-1,l-3} & \\ & C^{LL}_{l-1,l-2} \end{array}\right], \quad C^{LM}_{l+1,l} = \left[\begin{array}{c|c} & C^{UM}_{l-1,l-2} \\ \cline{2-2} 0_{l-1,l-4} & \\ & C^{LM}_{l-1,l-2} \end{array}\right], \quad C^{LR}_{l+1,l} = [0_{l,l-2}].$$

Let $L_{l+1,l}$ be the $m(l+1) \times m(l)$ matrix defined by the block matrix:

$$L_{l+1,l} = \left[\begin{array}{c|c} L^{UL}_{l+1,l} & L^{UR}_{l+1,l} \\ \hline L^{LL}_{l+1,l} & L^{LR}_{l+1,l} \end{array}\right]$$

where

$$\begin{aligned} L^{UL}_{l+1,l} &= \mathbb{A}^{UL}_{l+1,l} : \quad m(l) \times m(l-1) \text{ matrix}, \\ L^{UR}_{l+1,l} &= \left[\frac{0_{l-1,l-2}}{B_{l-2}}\right] : \quad m(l) \times m(l-2) \text{ matrix}, \\ L^{LL}_{l+1,l} &= \mathbb{A}^{LL}_{l+1,l} : \quad m(l-1) \times m(l-1) \text{ matrix}, \\ L^{LR}_{l+1,l} &= -I^t_{l-2,l-1} - C_{l-1,l-2} : \quad m(l-1) \times m(l-2) \text{ matrix}. \end{aligned}$$

We write down the above matrices for $l = 1, 2, 3, 4$.

$$L_{2,1} = \left[\begin{array}{c|c} 1 & \\ & 2 \\ \hline 2 & -3 \end{array}\right], \quad L_{3,2} = \left[\begin{array}{cc|c} 1 & & \\ & 1 & \\ 1 & -1 & 2 \\ \hline 1 & 1 & -3 \\ & 2 & -3 \end{array}\right], \quad L_{4,3} = \left[\begin{array}{ccc|cc} 1 & & & & \\ & 1 & & & \\ 1 & -1 & 1 & & \\ & 1 & -1 & 1 & 1 \\ & 1 & -1 & & 2 \\ \hline 1 & & 1 & -3 & \\ & 1 & 1 & -3 & \\ & & 2 & -2 & -1 \end{array}\right],$$

and

$$
L_{5,4} = \left[\begin{array}{ccccc|ccc}
1 & & & & & & & \\
 & 1 & & & & & & \\
1 & -1 & 1 & & & & & \\
 & 1 & -1 & 1 & & & & \\
 & 1 & -1 & & 1 & & & \\
 & & 1 & -1 & & 1 & & 1 \\
 & & 1 & -1 & & & 1 & 1 \\
 & & 1 & & -1 & 1 & -1 & 2 \\
\hline
1 & & & 1 & & -2 & -1 & \\
 & 1 & & 1 & & -2 & -1 & \\
 & & 1 & 1 & & -1 & -2 & \\
 & & & 1 & 1 & -1 & -1 & -1 \\
 & & & & 2 & -1 & -1 & -1
\end{array}\right].
$$

We define the elementary column operations on integer matrices to be:

(1) Multiply a column by -1,
(2) Add an integer multiple of one column to another column.

The elementary row operations are similarly defined. We know that the matrices $L_{l+1,l}$ is obtained from $\mathbb{A}_{l+1,l}$ by elementary column operations, that operation is denoted by Γ_l. The operation Γ_l is an $m(l) \times m(l)$ matrix corresponding to the column operation such that

$$
L_{l+1,l} = \mathbb{A}_{l+1,l}\Gamma_l.
$$

Since

$$
L_{l+1,l} = \left[\begin{array}{c|c}
\mathbb{A}^{UL}_{l+1,l} & \begin{array}{c} 0_{l-1,l-2} \\ \hline B_{l-2} \end{array} \\
\hline
\mathbb{A}^{LL}_{l+1,l} & -I^t_{l-2,l-1} - C_{l-1,l-2}
\end{array}\right],
$$

we may apply the elementary column operation $I_{l-1} \oplus \Gamma_{l-2}$ to $L_{l+1,l}$ so that the matrix B_{l-2} in $L_{l+1,l}$ goes to

$$
\left[\begin{array}{c|c}
\mathbb{A}^{UL}_{l-1,l-2} & \begin{array}{c} 0_{l-3,l-1} \\ \hline B_{l-4} \end{array}
\end{array}\right].
$$

The new matrix $L_{l+1,l}(I_{l-1} \oplus \Gamma_{l-2})$ is

$$
L_{l+1,l}(I_{l-1} \oplus \Gamma_{l-2}) = \left[\begin{array}{c|c}
\mathbb{A}^{UL}_{l+1,l} & \begin{array}{c} 0_{l-1,l-2} \\ \hline \begin{array}{c|c} \mathbb{A}^{UL}_{l-1,l-2} & \begin{array}{c} 0_{l-3,l-4} \\ \hline B_{l-4} \end{array} \end{array} \end{array} \\
\hline
\mathbb{A}^{LL}_{l+1,l} & (-I^t_{l-2,l-1} - C_{l-1,l-2})\Gamma_{l-2}
\end{array}\right].
$$

As

$$
B_{l-2n}\Gamma_{l-2n} = \left[\begin{array}{c|c}
\mathbb{A}^{UL}_{l-2n+1,l-2n} & \begin{array}{c} 0_{l-2n-1,l-2n-2} \\ \hline B_{l-2n-2} \end{array}
\end{array}\right]
$$

for $n = 1, 2, \ldots$ with $2n < l$, by continuing these procedures k-times for $l = 2k, 2k+1$ we finally get

$$B_2\Gamma_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & -1 & 2 \end{bmatrix} \quad \text{for } l = 2k \quad \text{and} \quad B_1\Gamma_1 = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \quad \text{for } l = 2k+1.$$

For $l = 2k, 2k+1$, let $\mathbb{M}_{l+1,l}$ be the $m(l+1) \times m(l)$ matrix obtained from $L_{l+1,l}$ after the k times procedures above. Then we have

$$\mathbb{M}_{l+1,l}(i,j) = \begin{cases} 0 & \text{if } i < j,\ 1 \le i, j \le m(l) \\ 1 & \text{if } i = j,\ 1 \le i < m(l) \\ 2 & \text{if } i = j = m(l). \end{cases}$$

Let $v_l = [v_l(i)]_{i=1}^{m(l-1)}$ be the column vector of length $m(l-1)$ defined by

$$v_l(i) = \mathbb{M}_{l+1,l}(m(l) + i, m(l)), \qquad i = 1, 2, \ldots, m(l-1)$$

so that the matrix $\mathbb{M}_{l+1,l}$ is of the form

$$\mathbb{M}_{l+1,l} = \left[\begin{array}{ccccc} 1 & & & & \\ & 1 & & & \\ & & \ddots & & \\ & * & & 1 & \\ & & & & 2 \\ \hline & & & & v_l(1) \\ & & & & v_l(2) \\ & & & & \vdots \\ & * & & & \\ & & & & v_l(m(l-1)) \end{array}\right].$$

For $l = 1, 2, 3, 4, 5, 6$, we see

$$v_1 = [-3], \quad v_2 = \begin{bmatrix} -3 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 3 \\ 3 \\ 1 \end{bmatrix}, \quad v_4 = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 1 \\ 1 \end{bmatrix}, \quad v_5 = \begin{bmatrix} -3 \\ -3 \\ -3 \\ -3 \\ -3 \\ -1 \\ -1 \\ -3 \end{bmatrix}, \quad v_6 = \begin{bmatrix} -3 \\ -3 \\ -3 \\ -3 \\ -3 \\ -3 \\ -3 \\ -3 \\ -1 \\ -1 \\ -1 \\ -3 \\ -3 \end{bmatrix}.$$

By induction, one has:

Lemma 3.5.

(i) $v_l(i) = \begin{cases} -3 & \text{if } l = 4k+1, 4k+2, k \in \mathbb{Z}_+, \text{ and } 1 \le i \le m(l-2), \\ 3 & \text{if } l = 4k+3, 4k+4, k \in \mathbb{Z}_+, \text{ and } 1 \le i \le m(l-2), \end{cases}$

(ii) $v_l(m(l-2) + i) = \widehat{v_{l-2}(i)}$ *for* $i = 1, 2, \ldots, m(l-3)$

where for $u = \pm 3, \pm 1$, *the integer* $\hat{u}$ *is defined by*

$$\hat{u} = \begin{cases} u - 4 & \text{if } u = 3, 1, \\ u + 4 & \text{if } u = -3, -1. \end{cases}$$

$$
\mathbb{N}_{l+1,l}(i,j) = \begin{cases} 1 & \text{if } i = j,\ 1 \le i < m(l), \\ 2 & \text{if } i = j = m(l), \\ v_l(i - m(l)) & \text{if } i > m(l),\ j = m(l), \\ 0 & \text{otherwise,} \end{cases}
$$

$$
\mathbb{H}_{l+1,l}(i,j) = \begin{cases} 1 & \text{if } i = j,\ 1 \le i < m(l), \\ 2 & \text{if } i = j = m(l), \\ -1 & \text{if } i > m(l),\ j = m(l), \\ 0 & \text{otherwise.} \end{cases}
$$

We set $m(l+1) \times m(l)$ matrices $\mathbb{N}_{l+1,l}$ and $\mathbb{H}_{l+1,l}$ by setting For $l = 1, 2, 3, 4$, one sees

$$
\mathbb{N}_{2,1} = \left[\begin{array}{cc} 1 & \\ & 2 \\ \hline & -3 \end{array}\right], \quad
\mathbb{N}_{3,2} = \left[\begin{array}{ccc} 1 & & \\ & 1 & \\ & & 2 \\ \hline & & -3 \\ & & -3 \end{array}\right], \quad
\mathbb{N}_{4,3} = \left[\begin{array}{ccccc} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & 2 \\ \hline & & & & 3 \\ & & & & 3 \\ & & & & 1 \end{array}\right], \quad
\mathbb{N}_{5,4} = \left[\begin{array}{cccccccc} 1 & & & & & & & \\ & 1 & & & & & & \\ & & 1 & & & & & \\ & & & 1 & & & & \\ & & & & 1 & & & \\ & & & & & 1 & & \\ & & & & & & 1 & \\ & & & & & & & 2 \\ \hline & & & & & & & 3 \\ & & & & & & & 3 \\ & & & & & & & 3 \\ & & & & & & & 1 \\ & & & & & & & 1 \end{array}\right]
$$

and

$$
\mathbb{H}_{2,1} = \left[\begin{array}{cc} 1 & \\ & 2 \\ \hline & -1 \end{array}\right], \quad
\mathbb{H}_{3,2} = \left[\begin{array}{ccc} 1 & & \\ & 1 & \\ & & 2 \\ \hline & & -1 \\ & & -1 \end{array}\right], \quad
\mathbb{H}_{4,3} = \left[\begin{array}{ccccc} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & 2 \\ \hline & & & & -1 \\ & & & & -1 \\ & & & & -1 \end{array}\right], \quad
\mathbb{H}_{5,4} = \left[\begin{array}{cccccccc} 1 & & & & & & & \\ & 1 & & & & & & \\ & & 1 & & & & & \\ & & & 1 & & & & \\ & & & & 1 & & & \\ & & & & & 1 & & \\ & & & & & & 1 & \\ & & & & & & & 2 \\ \hline & & & & & & & -1 \\ & & & & & & & -1 \\ & & & & & & & -1 \\ & & & & & & & -1 \\ & & & & & & & -1 \end{array}\right].
$$

By elementary row operations compatible to $I^t_{l,l+1}$, one gets the matrix $\mathbb{N}_{l+1,l}$ from the matrix $\mathbb{M}_{l+1,l}$. In the matrix $\mathbb{N}_{l+1,l}$, for $i = 1, 2, \dots, m(l)$, if $v_l(i) = -3$, then add the $m(l)$-th row to the $i + m(l)$-th row at the $i + m(l)$-th row, if $v_l(i) = 3$, then subtract the twice of $m(l)$-th row from the $i + m(l)$-th row at the $i + m(l)$-th row, if $v_l(i) = -3$, then subtract the $m(l)$-th row from the $i + m(l)$-th row at the $i + m(l)$-th row, then one gets the matrix $\mathbb{H}_{l+1,l}$. These row operations are compatible to the map $I^t_{l,l+1}$ and the relations

$$
I^t_{l,l+1}\mathbb{N}_{l,l-1} = \mathbb{N}_{l+1,l} I^t_{l-1,l}, \qquad I^t_{l,l+1}\mathbb{H}_{l,l-1} = \mathbb{H}_{l+1,l} I^t_{l-1,l}
$$

for $l = 2, 3, \dots$ hold. As

$$
(M^t_{l,l+1} - I^t_{l,l+1})\mathbb{Z}^{m(l)} = \mathbb{A}_{l+1,l}\mathbb{Z}^{m(l)} = L_{l+1,l}\mathbb{Z}^{m(l)} = \mathbb{M}_{l+1,l}\mathbb{Z}^{m(l)}, \qquad l \in \mathbb{N}
$$

we see that $\mathbb{Z}^{m(l+1)}/(M^t_{l,l+1} - I^t_{l,l+1})\mathbb{Z}^{m(l)}$ coincides with the group $\mathbb{Z}^{m(l+1)}/\mathbb{M}_{l+1,l}\mathbb{Z}^{m(l)}$ for all $l \in \mathbb{N}$. We then have

Proposition 3.6. *There exist isomorphisms*

$$
\xi_l : \mathbb{Z}^{m(l)}/\mathbb{M}_{l,l-1}\mathbb{Z}^{m(l-1)} \to \mathbb{Z}^{m(l)}/\mathbb{N}_{l,l-1}\mathbb{Z}^{m(l-1)},
$$
$$
\eta_l : \mathbb{Z}^{m(l)}/\mathbb{N}_{l,l-1}\mathbb{Z}^{m(l-1)} \to \mathbb{Z}^{m(l)}/\mathbb{H}_{l,l-1}\mathbb{Z}^{m(l-1)}
$$

of abelian groups such that the following diagrams are commutative:

$$\begin{array}{ccc}
\mathbb{Z}^{m(l)}/(M^t_{l-1,l} - I^t_{l-1,l})\mathbb{Z}^{m(l-1)} & \xrightarrow{\bar{I}^t_{l,l+1}} & \mathbb{Z}^{m(l+1)}/(M^t_{l,l+1} - I^t_{l,l+1})\mathbb{Z}^{m(l)} \\
\Big\| & & \Big\| \\
\mathbb{Z}^{m(l)}/\mathbb{M}_{l,l-1}\mathbb{Z}^{m(l-1)} & & \mathbb{Z}^{m(l+1)}/\mathbb{M}_{l+1,l}\mathbb{Z}^{m(l)} \\
\xi_l \Big\downarrow & & \xi_{l+1} \Big\downarrow \\
\mathbb{Z}^{m(l)}/\mathbb{N}_{l,l-1}\mathbb{Z}^{m(l)} & & \mathbb{Z}^{m(l+1)}/\mathbb{N}_{l+1,l}\mathbb{Z}^{m(l+1)} \\
\eta_l \Big\downarrow & & \eta_{l+1} \Big\downarrow \\
\mathbb{Z}^{m(l)}/\mathbb{H}_{l,l-1}\mathbb{Z}^{m(l)} & \xrightarrow{\widehat{I}^t_{l,l+1}} & \mathbb{Z}^{m(l+1)}/\mathbb{H}_{l+1,l}\mathbb{Z}^{m(l+1)}
\end{array}$$

where $\widehat{I}^t_{l,l+1} : \mathbb{Z}^{m(l)}/\mathbb{H}_{l,l-1}\mathbb{Z}^{m(l-1)} \to \mathbb{Z}^{m(l+1)}/\mathbb{H}_{l+1,l}\mathbb{Z}^{m(l)}$ *is the homomorphism induced by the matrix* $I^t_{l,l+1}$*. Hence we have an isomorphism*

$$K_0(\mathcal{O}_{\mathfrak{L}^{Ch(D_F)}}) \cong \varinjlim_l \{\widehat{I}^t_{l,l+1} : \mathbb{Z}^{m(l)}/\mathbb{H}_{l,l-1}\mathbb{Z}^{m(l-1)} \to \mathbb{Z}^{m(l+1)}/\mathbb{H}_{l+1,l}\mathbb{Z}^{m(l)}\}.$$

We fix $l \geq 3$. Define the $(m(l-1)+1) \times 1$ matrix R_{l-1} and the $(m(l-1)+1) \times (m(l-2)+1)$ matrix $I^R_{l-1,l-2}$ by setting:

$$R_{l-1} = \begin{bmatrix} 2 \\ -1 \\ \\ \vdots \\ -1 \end{bmatrix}, \qquad I^R_{l-1,l-2} = \left[\begin{array}{c|ccc} 1 & 0 & \dots & 0 \\ \hline 0 & & & \\ \vdots & & I_{l-2,l-1} & \\ 0 & & & \end{array}\right].$$

Then the following diagram is commutative:

$$\begin{array}{ccc}
\mathbb{Z}^{m(l)}/\mathbb{H}_{l,l-1}\mathbb{Z}^{m(l-1)} & \xrightarrow{\widehat{I}^t_{l,l+1}} & \mathbb{Z}^{m(l+1)}/\mathbb{H}_{l+1,l}\mathbb{Z}^{m(l)} \\
\Big\| & & \Big\| \\
\mathbb{Z}^{m(l-2)+1}/R_{l-2}\mathbb{Z} & \xrightarrow{\bar{I}^R_{l-1,l-2}} & \mathbb{Z}^{m(l-1)+1}/R_{l-1}\mathbb{Z}
\end{array}$$

where $\bar{I}^R_{l-1,l}$ is the homomorphism induced by the matrix $I^R_{l-1,l}$. Let $\varphi_{l-2} : \mathbb{Z}^{m(l-2)+1} \to \mathbb{Z}^{m(l-2)+1}$ be an isomorphism defined by the operations on the row vectors of $\mathbb{Z}^{m(l-2)+1}$ to add the 2-times multiplication of the second row to the first row, and subtract the second row from the k-th rows for $k = 3, 4, \dots, m(l-2)+1$. It is given by the matrix:

$$Q_{l-2} = \begin{bmatrix} 1 & 2 & & & \\ & 1 & & & \\ & -1 & 1 & & \\ & -1 & & 1 & \\ & \vdots & & & \ddots \\ & -1 & & & & 1 \end{bmatrix}.$$

Since $Q_{l-2}R_{l-2} = \begin{bmatrix} 0 \\ -1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$, φ_{l-2} yields an isomorphism

$$\varphi_{l-2} : \mathbb{Z}^{m(l-2)+1}/R_{l-2}\mathbb{Z} \to \mathbb{Z} \oplus 0 \oplus \mathbb{Z}^{m(l-2)-1} = \mathbb{Z}^{m(l-2)}.$$

Let $J_{l-1,l-2} : \mathbb{Z}^{m(l-2)-1} \to \mathbb{Z}^{m(l-1)-1}$ be a homomorphism defined by the $(m(l-1)-1) \times (m(l-2)-1)$ matrix

$$J_{l-1,l-2}(i,j) = \begin{cases} 0 & \text{if } i = 1, \\ I_{l-2,l-1}(i+1,j+1) & \text{if } i = 2, \dots, m(l-2)-1 \end{cases}$$

for $i = 1,2,\dots,m(l-1)-1$, $j = 1,2,\dots,m(l-2)-1$. We set $\widetilde{I}_{l-1,l-2} : \mathbb{Z}^{m(l-2)} \to \mathbb{Z}^{m(l-1)}$ a homomorphism defined by the $m(l-1) \times m(l-2)$ matrix

$$\widetilde{I}_{l-1,l-2}(i,j) = \begin{cases} 1 & \text{if } i = j = 1, \\ 0 & \text{if } i = 1,\ j \geq 2, \\ 0 & \text{if } i = 2, \\ I_{l-2,l-1}(i,j) & \text{if } i = 3,4,\dots,m(l-2)-1 \end{cases}$$

for $i = 1,2,\dots,m(l-1)$, $j = 1,2,\dots,m(l-2)$. That is,

$$\widetilde{I}_{l-1,l-2} = \left[\begin{array}{c|ccc} 1 & 0 & \dots & 0 \\ \hline 0 & & & \\ \vdots & & J_{l-1,l-2} & \\ 0 & & & \end{array}\right].$$

Lemma 3.7. *The diagram*

$$\begin{array}{ccc} \mathbb{Z}^{m(l-2)+1}/R_{l-2}\mathbb{Z} & \xrightarrow{\bar{I}^R_{l-1,l-2}} & \mathbb{Z}^{m(l-1)+1}/R_{l-1}\mathbb{Z} \\ {\scriptstyle \varphi_{l-2}}\downarrow & & {\scriptstyle \varphi_{l-1}}\downarrow \\ \mathbb{Z}^{m(l-2)} & \xrightarrow{\widetilde{I}_{l-1,l-2}} & \mathbb{Z}^{m(l-1)} \end{array}$$

is commutative. Hence we have an isomorphism

$$K_0(\mathcal{O}_{\mathfrak{L}^{Ch(D_F)}}) \cong \mathbb{Z} \oplus \varinjlim_{l} \{J_{l-1,l-2} : \mathbb{Z}^{m(l-2)-1} \to \mathbb{Z}^{m(l-1)-1}\}.$$

Proof. Since the commutativity $\varphi_{l-1} \circ \bar{I}^R_{l-1,l-2} = \widetilde{I}_{l-1,l-2} \circ \varphi_{l-2}$ is immediate, one has

$$K_0(\mathcal{O}_{\mathfrak{L}^{Ch(D_F)}}) \cong \varinjlim_{l} \{\widetilde{I}_{l-1,l-2} : \mathbb{Z}^{m(l-2)} \to \mathbb{Z}^{m(l-1)}\}.$$

As $\widetilde{I}_{l-1,l-2} = 1 \oplus J_{l-1,l-2}$, the assertion is clear. □

We will compute the group of the inductive limit $\varinjlim_{l} \{J_{l+1,l} : \mathbb{Z}^{m(l)-1} \to \mathbb{Z}^{m(l+1)-1}\}$, that we denote by G. Let $I^c_{l+1,l}$ be the $(m(l)-2) \times (m(l)-1)$ matrix defined by

$$I^c_{l+1,l}(i,j) = I_{l,l+1}(i+2,j+1) \quad \text{for} \quad i = 1,\dots,m(l)-2,\ j = 1,\dots,m(l)-1.$$

Hence $J_{l+1,l} = \left[\begin{array}{c} 0 \ \cdots \ 0 \\ \hline I^c_{l+1,l} \end{array}\right]$. It gives rise to a homomorphism :

$$I^c_{l+1,l} : \mathbb{Z}^{m(l)-1} \to \mathbb{Z} \oplus I^c_{l+1,l}\mathbb{Z}^{m(l)-1} \subset \mathbb{Z}^{m(l)-1}.$$

Put

$$\mathbb{Z}(l) = \mathbb{Z} \oplus \cdots \oplus \mathbb{Z} = \mathbb{Z}^{m(l)-1}.$$

For $k \in \mathbb{N}$, take $l \in \mathbb{Z}_+$ such that $k \le m(l)$. Define a sequence of positive integers

$$g_k = \sum_{j=1}^{k} \sum_{i=2}^{m(l+1)} I^t_{l,l+1}(i,j), \qquad k = 1, 2, \dots$$

that is independent of the choice of l, so that

$$g_1 = 1, \quad g_2 = 2, \quad g_3 = 4, \quad g_4 = 6, \quad g_5 = 7, \quad g_6 = 9, \dots.$$

Define for $l \ge k$,

$$\mathbb{Z}(l;k) = \overbrace{0 \oplus \cdots \oplus 0}^{g_k} \oplus \overbrace{\mathbb{Z} \oplus \cdots \oplus \mathbb{Z}}^{m(l)-1-g_k} \subset \mathbb{Z}^{m(l)-1} = \mathbb{Z}(l)$$

so that we have

$$I^c_{l+1,l}(\mathbb{Z}(l;k)) \subset \mathbb{Z}(l+1;k+1).$$

Set the group of the inductive limit

$$G_k = \varinjlim_n \{I^c_{k+n+1,k+n} : \mathbb{Z}(k+n;n) \to \mathbb{Z}(k+n+1;n+1)\}.$$

Since the following diagram is commutative:

$$\begin{array}{ccccccccccc}
\mathbb{Z}(1) & \xrightarrow{I^c_{2,1}} & \mathbb{Z}(2;1) & \xrightarrow{I^c_{3,2}} & \mathbb{Z}(3;2) & \xrightarrow{I^c_{4,3}} & \mathbb{Z}(4;3) & \xrightarrow{I^c_{5,4}} & \cdots & \longrightarrow & G_1 \\
 & & \downarrow{\scriptstyle \iota} & & \downarrow{\scriptstyle \iota} & & \downarrow{\scriptstyle \iota} & & & & \\
 & & \mathbb{Z}(2) & \xrightarrow{I^c_{3,2}} & \mathbb{Z}(3;1) & \xrightarrow{I^c_{4,3}} & \mathbb{Z}(4;2) & \xrightarrow{I^c_{5,4}} & \cdots & \longrightarrow & G_2 \\
 & & & & \downarrow{\scriptstyle \iota} & & \downarrow{\scriptstyle \iota} & & & & \\
 & & & & \mathbb{Z}(4) & \xrightarrow{I^c_{4,3}} & \mathbb{Z}(4;1) & \xrightarrow{I^c_{5,4}} & \cdots & \longrightarrow & G_3 \\
 & & & & & & \downarrow{\scriptstyle \iota} & & & & \\
 & & & & & & \mathbb{Z}(4) & \xrightarrow{I^c_{5,4}} & \cdots & \longrightarrow & G_4 \\
 & & & & & & & & \vdots & &
\end{array}$$

where the vertical arrows ι mean the natural inclusion maps, one sees the next lemma:

Lemma 3.8.

(i) *For each $k = 1, 2, \dots$, the group G_k is isomorphic to the abelian group $C(\mathfrak{K}_k, \mathbb{Z})$ of all integer valued continuous functions on a Cantor discontinuum $\mathfrak{K}_k$.*

(ii) *The sequence $G_k, k = 1, 2, \dots$ are increasing whose union generate G.*

Hence one has

Lemma 3.9. *The group G is isomorphic to the countable direct sum of the group $C(\mathfrak{K}, \mathbb{Z})$ of all integer valued continuous functions on a Cantor discontinuum $\mathfrak{K}$.*

Proof. It is easy to see that G_k is isomorphic to the direct sum $C(\mathfrak{K}_{k,k-1},\mathbb{Z})\oplus G_{k-1}$ of all integer valued continuous functions on a Cantor discontinuum $\mathfrak{K}_{k,k-1}$ and G_{k-1} for each k. Hence we have

$$\begin{aligned} G_k &\cong C(\mathfrak{K}_{k,k-1},\mathbb{Z})\oplus G_{k-1} \\ &\cong C(\mathfrak{K}_{k,k-1},\mathbb{Z})\oplus C(\mathfrak{K}_{k-1,k-2},\mathbb{Z})\oplus\cdots\oplus C(\mathfrak{K}_{2,1},\mathbb{Z})\oplus G_1. \end{aligned}$$

Since both G_1 and $C(\mathfrak{K}_{i,i-1},\mathbb{Z})$ are isomorphic to the group $C(\mathfrak{K},\mathbb{Z})$ of all integer valued continuous functions on a Cantor discontinuum $\mathfrak{K}$, we have

$$G \cong \varinjlim_k G_k \cong C(\mathfrak{K},\mathbb{Z})^\infty.$$

□

Therefore we conclude

Theorem 3.10.

$$K_0(\mathcal{O}_{\mathfrak{L}^{Ch(D_F)}}) \cong \mathbb{Z}\oplus C(\mathfrak{K},\mathbb{Z})^\infty, \qquad K_1(\mathcal{O}_{\mathfrak{L}^{Ch(D_F)}}) \cong 0.$$

Proof. Since $K_0(\mathcal{O}_{\mathfrak{L}^{Ch(D_F)}})$ is isomorphic to

$$\mathbb{Z}\oplus \varinjlim_l \{J_{l+1,l} : \mathbb{Z}^{m(l)-1} \to \mathbb{Z}^{m(l+1)-1}\}$$

and the second summand above denoted by G is isomorphic to $C(\mathfrak{K},\mathbb{Z})^\infty$, one gets $K_0(\mathcal{O}_{\mathfrak{L}^{Ch(D_F)}}) \cong \mathbb{Z}\oplus C(\mathfrak{K},\mathbb{Z})^\infty$. We have already seen the formula $K_1(\mathcal{O}_{\mathfrak{L}^{Ch(D_F)}}) \cong 0$. □

Therefore Theorem 1.1 holds.

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